Method of Gauss-Jordan Elimination for classroom use

A special case of ordinary row-reduction by Gauss-Jordan Elimination developed by Dick Furnas

Start–

Consider an integer matrix:

$$\left[\begin{array}{ccc|c} 2 & 4 & 0 & 6 \\ 2 & 1 & 3 & 5 \\ 0 & 1 & 1 & -3 \end{array}\right]\begin{array}{c} /2 \\ \\ \end{array}$$

Simplify:

Divide so that no row has a common factor.

Setup:

Multiply to achieve the Lowest Common Multiple (L.C.M.) throughout the column: Make a positive coefficient on the element to be kept, a negative coefficient on the elements to be zeroed.

Add:

Copy the row to be kept.
Add it to each of the other rows
(*You will never need to subtract.*)

Work on Column 1
L.C.M. = 2

Simplify:

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 2 & 1 & 3 & 5 \\ 0 & 1 & 1 & -3 \end{array}\right]\begin{array}{c} *2 \\ *\text{-}1 \\ *1 \end{array}$$

Setup:

$$\left[\begin{array}{ccc|c} +2 & 4 & 0 & 6 \\ -2 & -1 & -3 & -5 \\ 0 & 1 & 1 & -3 \end{array}\right]$$

Add:

$$\left[\begin{array}{ccc|c} 2 & 4 & 0 & 6 \\ 0 & 3 & -3 & 1 \\ 0 & 1 & 1 & -3 \end{array}\right]\begin{array}{c} /2 \\ /1 \\ /1 \end{array}$$

Work on Column 2
L.C.M. = 6

Simplify:

$$\left[\begin{array}{ccc|c} 1 & 2 & 0 & 3 \\ 0 & 3 & -3 & 1 \\ 0 & 1 & 1 & -3 \end{array}\right]\begin{array}{c} *\text{-}3 \\ *\ 2 \\ *\text{-}6 \end{array}$$

Setup:

$$\left[\begin{array}{ccc|c} -3 & -6 & 0 & -9 \\ 0 & +6 & -6 & 2 \\ 0 & -6 & -6 & 18 \end{array}\right]$$

Add:

$$\left[\begin{array}{ccc|c} -3 & 0 & -6 & -7 \\ 0 & 6 & -6 & 2 \\ 0 & 0 & -12 & 20 \end{array}\right]\begin{array}{c} /\text{-}1 \\ /2 \\ /\text{-}4 \end{array}$$

Work on Column 3
L.C.M. = 6

Simplify:

$$\left[\begin{array}{ccc|c} 3 & 0 & 6 & 7 \\ 0 & 3 & -3 & 1 \\ 0 & 0 & 3 & -5 \end{array}\right]\begin{array}{c} *\text{-}1 \\ *2 \\ *2 \end{array}$$

Setup:

$$\left[\begin{array}{ccc|c} -3 & 0 & -6 & -7 \\ 0 & 6 & -6 & 2 \\ 0 & 0 & +6 & -10 \end{array}\right]$$

Add:

$$\left[\begin{array}{ccc|c} -3 & 0 & 0 & -17 \\ 0 & 6 & 0 & -8 \\ 0 & 0 & 6 & -10 \end{array}\right]\begin{array}{c} /\text{-}1 \\ /2 \\ /2 \end{array}$$

End.

Divide to make lead entries = 1

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 17/3 \\ 0 & 1 & 0 & -4/3 \\ 0 & 0 & 1 & -5/3 \end{array}\right] \longleftarrow \left[\begin{array}{ccc|c} 3 & 0 & 0 & 17 \\ 0 & 3 & 0 & -4 \\ 0 & 0 & 3 & -5 \end{array}\right]\begin{array}{c} /3 \\ /3 \\ /3 \end{array}$$

Virtues of the method:

- Utterly deterministic – no confusion over what to do next.
- Self-documenting.
- Simple arithmetic:
  - Uses only + , * and integer divide– no fractions until the end.
  - Uses smallest possible integers often with known factors.
- Easy to follow, correct, or resume if interrupted.
- Minimizes errors when calculating by hand or on the board.