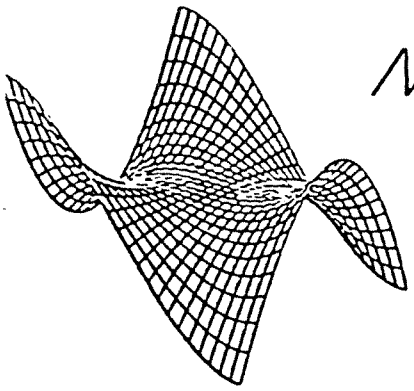
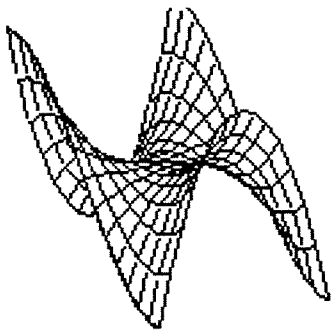


Mathematics Support Capsules

Basic Algebra Review

These Mathematics Support Capsules were prepared under the supervision of Beverly West, Lecturer, Department of Mathematics, Cornell University, Ithaca, New York 14853, with assistance from King Chan, Ken Gardner, Kristen Jackson, Ann Michel, Ken Okamoto, and funding from the Exxon Corporation. The authors solicit your comments, corrections, and suggestions for future revision. August, 1981



Mathematics Support Capsules

BASIC ALGEBRA
0. DIAGNOSTIC

Combine and simplify as much as possible the following expressions:

Questions	*Answers*
1) $\dfrac{1}{a+b} - \dfrac{2a}{a^2-b^2}$	1)_____
2) $\dfrac{x^2+2x+1}{2x^2} \div \dfrac{x+1}{x+2}$	2)_____
3) $-\dfrac{a+b}{ac+bd}$	3)_____
4) $\dfrac{(2a)^3}{a^5}$	4)_____
5) $(0.2a^2)^4$	5)_____
6) $\dfrac{8y^n}{-2y^{n-1}}$	6)_____

7) $\sqrt[3]{-64y^{27}}$ 7)_____

8) $\sqrt{a^2+b^2}$ 8)_____

9) $(a+b)^3$ 9)_____

10) $(\sqrt{x}+3\sqrt{y})(\sqrt{x}-\sqrt{y})$ 10)_____

Solve the following equations for x:

11) $x^3-x^2-6x=0$ 11)_____

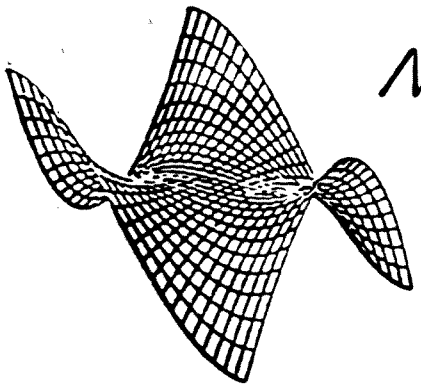
12) $x^2+7x=-3$ 12)_____

Now check your answers on the next page!

Algebra Diagnostic Answers		
Answers to algebra diagnostic test	If you missed these questions, review the indicated sections of the MSC *Algebra* Capsule.	If you need more than a brief review, work through the indicated sections of Hughes-Hallett, *Algebra.*
1) $\dfrac{-a-b}{a^2-b^2} = \dfrac{-1}{a-b} = \dfrac{1}{b-a}$ 2) $\dfrac{x^2+3x+2}{2x^2}$ 3) cannot be simplified	II. Fractions	Chapters 4, 10
4) $\dfrac{8}{a^2}$ 5) $0.0016a^8$ 6) $-4y$	III. Exponents	Chapter 7 (using 10^m, but the rules apply to a^m)
7) $-4y^9$ 8) cannot be simplified	IV. Radicals	Chapter 6
9) $a^3+3a^2b+3ab^2+b^3$ 10) $x+2\sqrt{xy}-3y$	V. Simplifying Algebraic Expressions	Chapters 3, 8, 9, 11
11) $x = 0, 3, -2$ 12) $x = \dfrac{-7 \pm \sqrt{37}}{2}$	VI. Solving Equations	Chapters 12, 14, 15

If you missed many questions, try reading Chapters 1, 4, and 8 of Isaac Asimov, *Realm of Algebra* (Fawcett Publications, 1961), which is a brief, conversational paperback that many students have found helpful (especially Chapters 1 and 8), or go directly to Deborah Hughes-Hallett, *The Math Workshop: Algebra* (W. W. Norton, 1980), which is also very conversational, but detailed, with excellent exercises, or George F. Simmons, *Precalculus Mathematics in a Nutshell* (William Kaufmann, 1981), a delightfully succinct paperback covering essentials.

All these books are available for examination or browsing in the Mathematics Support Center and for sale at the Campus Bookstore and other bookstores in Collegetown.



Mathematics Support Capsules

BASIC ALGEBRA

I. REFERENCE

90% of mathematics is involved with taking a number and rewriting it in a more useful form. Two fairly elementary (but often useful) things to do to a number to change its form but not its value are:

1) add zero to it

2) multiply it by one

Some more exciting ways of changing the "form" of a number but not its "content" are summarized in the following rules: (here a, b, and c represent numbers).

3) $a + b = b + a$ } (commutative laws)

4) $ab = ba$ }

5) $(a+b) + c = a + (b+c)$ } (associative laws)

6) $(ab)c = a(bc)$ }

7) $(a+b)c = ac + bc$ (distributive laws)

Dealing with <u>signs</u>: Remember that $(-a)$ may be thought of as $(-1)a$, so you need only remember

$$(+1)(-1) = (-1)(+1) = -1$$

$$(-1)(-1) = +1$$

$$\frac{(+1)}{(-1)} = \frac{(-1)}{(+1)} = -1$$

$$\frac{(-1)}{(-1)} = +1$$

Terminology: <u>Factors</u> are quantities connected by <u>multiplication</u>.
<u>Terms</u> are quantities connected by <u>addition</u>.

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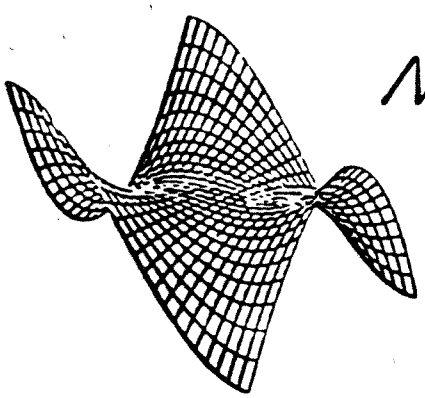

Mathematics Support Capsules

BASIC ALGEBRA
II. FRACTIONS

1. Adding: You can only add fractions with the same denominator.

If you are adding fractions with different denominators, change the denominators so that they are the same, while multiplying by one so as not to change their values.

Example: To add $\frac{1}{6}+\frac{1}{5}$ you multiply the first fraction by $\frac{5}{5}$ and the second by $\frac{6}{6}$, so that both denominators are 30.

$$\frac{1}{6}\cdot\frac{5}{5}+\frac{1}{5}\cdot\frac{6}{6}=\frac{5}{30}+\frac{6}{30}$$

Now these fractions can be added to give the answer: $\frac{11}{30}$.

In general, any time you add two fractions with different denominators, follow the same method:

$$\boxed{\frac{a}{b}+\frac{c}{d}=}\ \frac{a}{b}\left(\frac{d}{d}\right)+\frac{c}{d}\left(\frac{b}{b}\right)=\frac{ad}{bd}+\frac{cb}{db}\ \boxed{=\frac{ad+cb}{bd}}$$

multiplying by one — same denominators

Example: $\frac{1}{xy}+\frac{1}{z}=\frac{1}{xy}\cdot\frac{z}{z}+\frac{1}{z}\cdot\frac{xy}{xy}=\frac{z}{xyz}+\frac{xy}{xyz}=\frac{z+xy}{xyz}$

2. Multiplying: To multiply fractions, multiply the numerators together and multiply the denominators together.

Example: $\frac{2}{5}\cdot\frac{5}{2} = \frac{10}{10} = 1$

In general terms, $\boxed{\frac{a}{b}\cdot\frac{c}{d} = \frac{ac}{bd}}$

3. Dividing: To divide two fractions, invert the divisor and multiply.

Example: $\frac{\frac{4}{3}}{\frac{7}{5}} = \frac{4}{3}\cdot\frac{5}{7} = \frac{20}{21}$

flip the divisor

or,

$$\frac{4}{3} \div \frac{7}{5} = \frac{4}{3}\cdot\frac{5}{7} = \frac{20}{21}$$

Example: $\frac{\frac{4}{y}}{x} = \frac{\frac{4}{y}}{\frac{x}{1}} = \frac{4}{y}\cdot\frac{1}{x} = \frac{4}{yx}$

So, the general rules are

$$\boxed{\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b}\cdot\frac{d}{c} = \frac{ad}{bc}}$$

$$\boxed{\frac{a}{b} \div \frac{c}{d} = \frac{a}{b}\cdot\frac{d}{c} = \frac{ad}{bc}}$$

4. Cancelling: As in the case with numbers, we may cancel factors that are variables, because a number multiplied or divided by one is unchanged.

Examples: $\frac{15}{10} = \frac{5\cdot 3}{5\cdot 2} = \underbrace{\frac{\cancel{5}}{\cancel{5}}}_{=1}\cdot\frac{3}{2} = \frac{3}{2}$

$$\frac{x^2yz}{xz} = \frac{\cancel{x}\cdot x\cdot y\cancel{z}}{\cancel{x}\cancel{z}} = xy$$

In general,

$$\boxed{\frac{ac}{bc} = \frac{a}{b}}$$

Note, however, that c must be a factor of the entire numerator and the entire denominator.

Example: $\frac{a+b}{a}$ cannot be simplified by cancellation

$\frac{\cancel{a}+b}{\cancel{a}}$ is not legitimate, because it changes the value of the number.

5. You cannot divide by zero.

$\frac{x}{0}$ is not a real number.

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Exercises:

1. $\frac{3}{x} - \frac{x}{6} =$

2. $\frac{-17}{14x} + \frac{b}{2x} =$

3. $\frac{x}{7(x+1)} + \frac{2}{3x} =$

4. $(\frac{3x}{x-1}) \cdot (\frac{x^2}{x+1}) =$

5. $(\frac{3x}{x-1}) \div (\frac{x^2}{x+1}) =$

6. $\frac{ab^2}{c} + \frac{b^2}{a^2c} + \frac{2}{b} =$

Simplify if possible

7. $\frac{ab + ac}{ad} =$

8. $\frac{xy}{x + y} =$

9. $\frac{x^2 + 2x + 1}{x^2 - 1} =$

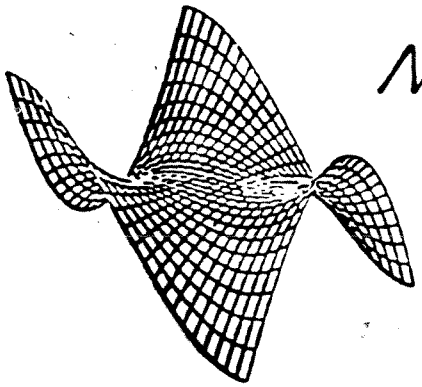
Answers: Your answers should be equivalent to these.

1) $\frac{18-x^2}{6x}$ 2) $\frac{-17 + 7b}{14x}$ 3) $\frac{3x^2 + 14x + 14}{21x\ (x+1)}$

4) $\frac{3x^3}{x^2-1}$ 5) $\frac{3x}{x-1} \cdot \frac{x + 1}{x^2} = \frac{3(x+1)}{x(x-1)} = \frac{3x+3}{x^2-x}$

6) $\frac{a^3b^3 + b^3 + 2a^2c}{a^2bc}$ 7) $\frac{b+c}{d}$ 8) <u>cannot</u> be simplified

9) $\frac{(x+1)^2}{(x+1)(x-1)} = \frac{x+1}{x-1}$



Mathematics Support Capsules

BASIC ALGEBRA

III. EXPONENTS

If n is a positive whole number, then

a^n means $a \cdot a \cdot a \cdots a$ (n factors)

i.e., a product of n (a)'s

So: what can we do with them?

in general, the laws of exponents are:

$x^3 \cdot x^2 =$ ____?

$(= (x \cdot x \cdot x)(x \cdot x) = x^5)$

$a^m a^n = a^{m+n}$

$(x^3)^2 =$ ____?

$(= (x \cdot x \cdot x) \cdot (x \cdot x \cdot x) = x^6)$

$(a^m)^n = a^{mn}$

$(xy)^2 =$ ____?

$(= (xy) \cdot (xy) = x^2 y^2)$

$(ab)^n = a^n b^n$

(Note: there is not much we can do with $a^m b^n$ unless $a=b$ or $m=n$)

And, in order that the laws of exponents extend to exponents that are not positive whole numbers, we make the following definitions:

$a^{-n} \equiv \frac{1}{a^n}$ $\quad 8^{-2} =$ _____? $\quad (\frac{1}{64})$

$a^{1/n} \equiv \sqrt[n]{a}$ $\quad 8^{1/3} =$ _____. $\quad (2)$

therefore

$\frac{a^m}{a^n} = a^{m-n}$ $\quad \frac{8^2}{8^5} =$ _____? $\quad (\frac{1}{512})$

$a^{m/n} = \sqrt[n]{a^m}$ $\quad 8^{2/3} =$ _____? $\quad (4)$

Note that as a result of these definitions,

$\boxed{a^0 = 1}$ for any $a \neq 0$.

Example of exponent simplification: (try it yourself first - there are many proper routes to the right answer.)

$$\frac{(\frac{1}{3}a^2b)^3}{\frac{1}{9}ab^2} = \text{\_\_\_\_\_}?$$

$$\left(\frac{\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}(a\cdot a)\cdot(a\cdot a)\cdot(a\cdot a)b\cdot b\cdot b}{\frac{1}{3}\cdot\frac{1}{3}\cdot a\cdot b\cdot b} = \frac{1}{3}a^5b\right)$$

Exercises:

1) $c^3(-3c^4) =$

2) $c^3(-3c)^4 =$

3) $\dfrac{-x^2y}{\frac{1}{3}xy} =$

4) $\dfrac{9r^2t^n}{-3r^nt^2} =$

5) $\dfrac{0.6\,a^nb^k}{0.3\,a^{-n}b^{k-1}} =$

6) $(2a^2bc^3)^{13} =$

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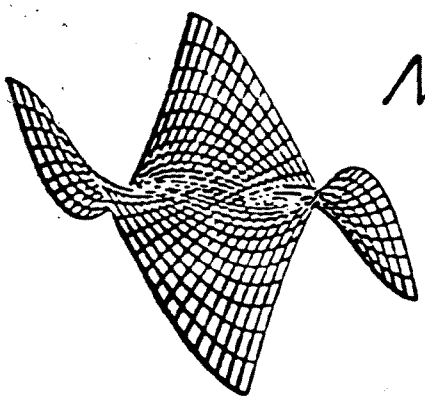
Answers:

1) $-3c^7$ 2) $81c^7$ 3) $-3x$ 4) $-3r^{2-n}t^{n-2}$

$2a^{2n}b$ 6) $8192a^{26}b^{13}c^{39}$



Mathematics Support Capsules

BASIC ALGEBRA
IV. RADICALS

Taking the root of a number, which is indicated by radicals ($\sqrt{\ }$), is the opposite of raising a number to a power, which is indicated by exponents.

		exponential	radical
Examples:	since	$5^2 = 25$	$\sqrt{25} = 5$
	since	$2^3 = 8$	$\sqrt[3]{8} = 2$

Cautions: (to be used as guidelines)

1) 25 has two square roots, +5 and -5, because $25 = (-5)^2$ also. Possible confusion is eliminated by the convention that:

> the $\sqrt{\ }$ symbol always indicates the positive square root

thus $\sqrt{25} = 5$, and the negative square root is written as $-\sqrt{25} = -5$.

2) When working only with real numbers, you cannot take an even root of a negative number. (E.g. $\sqrt{-1}$ is an imaginary or complex number, which you will study later.) So if you see $\sqrt{\text{negative number}}$, you do not have a real number. Either the problem does not have a real solution, or somebody has made a mistake. Check for mistakes in your calculations, or typos in the book.

Note: $\sqrt[3]{-8} = -2$ because $(-2)(-2)(-2) = -8$.

3) Taking roots of a number involves factors. ($\sqrt{25} = 5$ because $25 \cdot 5 \times 5$ so $\sqrt{25} = \sqrt{5 \times 5}$.

Radicals consider everything under the $\sqrt{\ }$ sign as a whole; the only way it can be considered in parts is as factors, not terms.

A common mistake made by students is to say that $\sqrt{a^2 + b^2}$ equals $a + b$. (Substitute the values $a = 3$ and $b = 4$ to see for yourself that this is false.)

4) You can only multiply radicals if the root number (i.e., $\sqrt[5]{2}$ ← root number) is the same. So $\sqrt{a}\cdot\sqrt{b} = \sqrt{ab}$, but $\sqrt[3]{a}\cdot\sqrt{b} = \sqrt[3]{a}\cdot\sqrt{b}$. To simplify $\sqrt[3]{a}\cdot\sqrt{b}$, find the common root number. In this case it is 6. $\sqrt[3]{a} = \sqrt[6]{a^2}$ and $\sqrt{b} = \sqrt[6]{b^3}$, so we have $\sqrt[6]{a^2}\cdot\sqrt[6]{b^3} = \sqrt[6]{a^2b^3}$. The same applies with dimension $\frac{\sqrt[3]{a}}{\sqrt{b}} = \sqrt[3]{\frac{a}{b}}$.

Example: $\sqrt{9x^2} = \sqrt{9}\cdot\sqrt{x^2} = 3x$.

Exercises:

1) $\sqrt[3]{343} =$

2) $\sqrt{192\ x^3y^2}$

3) $\sqrt{900\ x^5}$

4) Solve for x & $\sqrt{x+1} = 3$

5) $\sqrt[3]{x^3+3x^2+3x+1}$

Solutions:

1) $\sqrt[3]{343} = \sqrt[3]{7\cdot 7\cdot 7} = 7$

2) $\sqrt{192\ x^3y^2} = \sqrt{3\cdot 64\ x\cdot x^2\cdot y^2} = 8xy\sqrt{3x}$

3) $\sqrt{900\ x^5} = \sqrt{25\cdot 36\ x^5} = 5\cdot 6\sqrt{x^5} = 30\sqrt{x^4\cdot x} = 30\ x^2\sqrt{x}$

4) $\sqrt{x+1} = 3$ square both sides $x+1 = 9 \Longrightarrow x = 8$

5) $\sqrt[3]{x^3+3x^2+3x+1}$ $\quad (x+1)\sqrt{x^3+3x^2+3x+1}$ $\quad x^2+2x+1 = (x+1)(x+1)$

so $\sqrt[3]{x^3+3x^2+3x+1} = \sqrt[3]{(x+1)(x+1)(x+1)} = (x+1)$

Rationalizing a denominator: If you wish to remove radicals from the denominator of a fraction, proceed as follows:

If the denominator has only one term, multiply top and bottom by the radical.

$$\frac{3}{\sqrt{5}} = \frac{3}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

If the denominator has two terms, $\sqrt{a} + \sqrt{b}$, multiply top and bottom by its conjugate, $\sqrt{a} - \sqrt{b}$. The conjugate is the same two terms with the opposite sign between. Then your new denominator will have no radicals. For instance,

$$\frac{5 + \sqrt{5}}{\sqrt{3} + 1} = \frac{5 + \sqrt{5}}{\sqrt{3} + 1} \cdot \frac{(\sqrt{3} - 1)}{(\sqrt{3} - 1)} = \frac{5\sqrt{3} + \sqrt{5}\cdot\sqrt{3} - 5 - \sqrt{5}}{3 + \sqrt{3} - \sqrt{3} - 1}$$

$$= 1$$

$$= \frac{5\sqrt{3} + \sqrt{15} - 5 - \sqrt{5}}{2}$$

Fractional Exponents:

A radical can also be expressed by fractional exponents.

The important connection is

$$\boxed{\sqrt[n]{a} = a^{1/n}}$$

Now all the rules for adding, multiplying, etc. of exponents apply. (See ALG III.)

Example: Solve:

1) $5^{1/3} \cdot 5^{1/2}$

$= 5^{(1/3 + 1/2)} = 5^{5/6}$

2) $(5^{1/3})^{1/2}$

$= 5^{1/6}$

Radicals and exponent notation are two equivalent ways of learning to work with radicals. Although you are probably more familiar with the first way ($\sqrt[n]{a}$), try to understand the ideas presented in the second way ($a^{1/n}$). Once you get the hang of it, the second way will be very simple. For instance, exponent notation makes it possible to multiply and divide radicals very easily.

Example:

Convert $\dfrac{\sqrt[3]{a^5}}{\sqrt{a}}$ to fractional exponent notation and simplify.

$$\sqrt[3]{a^5} = a^{5/3} \quad \text{and} \quad \sqrt{a} = a^{1/2}$$

so

$$\frac{\sqrt[3]{a^5}}{\sqrt{a}} = \frac{a^{5/3}}{a^{1/2}} \left(\text{recall } \frac{a^m}{a^n} = a^{m-n} \right)$$

so

$$= a^{(5/3 - 1/2)} = a^{(7/6)} = \sqrt[6]{a^7}$$ (See exercise # 1)

Exercises:

Simplify using fractional notation:

1) $\sqrt[6]{x^7} =$

2) $\sqrt[3]{x} \cdot \sqrt[6]{x^4} =$

Simplify:

3) $(x^{1/2})^4 =$

4) $(x^{1/3}y^{1/2} + x^{1/2}y^{1/3})^2 =$

Solutions:

1) $\sqrt[6]{x^7} = x^{7/6} = x^{(1+1/6)} = x \cdot x^{1/6} = x \cdot \sqrt[6]{x}$

2) $\sqrt[3]{x} \cdot \sqrt[6]{x^4} = x^{1/3} \cdot x^{4/6} = x^{(1/3+4/6)} = x^{(2/6+4/6)} = x$

Note: $\sqrt[3]{x} = \sqrt[6]{x^2} = \sqrt[9]{x}$ in fractional notation $x^{1/3} = x^{2/6} = x^{3/9}$

3) $(x^{1/2})^4 = x^{4/2} = x^2$

4) $(x^{1/3}y^{1/2} + x^{1/2}y^{1/3})^2 = (x^{1/3}y^{1/2} + x^{1/2}y^{1/3})(x^{1/3}y^{1/2} + x^{1/2}y^{1/3})$

$= (x^{2/3}y + x^{5/6}y^{5/6} + x^{5/6}y^{5/6} + xy^{2/3})$

$= x^{2/3}y + 2(xy)^{5/6} + xy^{2/3}$

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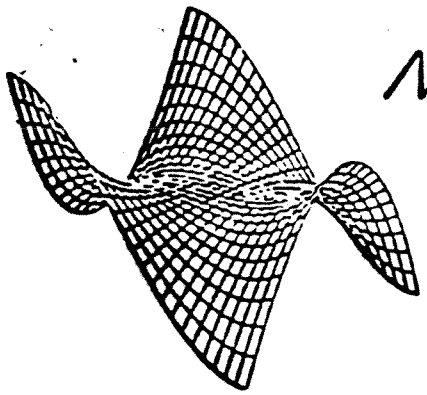

Mathematics Support Capsules

BASIC ALGEBRA

V. SIMPLIFYING ALGEBRAIC EXPRESSIONS

Rules to keep in mind:

1. In polynomials you can only combine like terms
   (e.g., $x^2y + 3xy^2 + 4x - xy^2 = x^2y + 2xy^2 + 4x$.)
2. Work first inside innermost parenthesis until they can be eliminated.
   (e.g.: $-(1-(1-3)^2) = -(1-(-2)^2) = -(1-4) = -(-3) = 3$.)
3. Within any parenthesis, if there is any confusion, do operations in the following order: Multiplication, Division, Addition, Subtraction
   (e.g. $(1-3\cdot4) = 1-12 = -11$) (= My Dear Aunt Sally)
4. Multiplying polynomials: you must multiply every term of first by every term of second.
   (e.g. $(a+bx)(x^2-x) = ax^2-ax+bx^3-bx^2 = bx^3+(a-b)x^2-ax$)

Exercises:

Simplify:

1) $(x^2-2y)(x^2-5y)$

2) $(m^2-mn+n^2)(m^2+mn+n^2)$

If $A = x^2-xy+y^2$, $B = 3x-4y$, $C = 2x+y$, compute

3) $A-BC$

4) B^2-C^2

Solutions:

1) $x^4-7x^2y+10y^2$

2) $m^4+m^2n^2+n^4$

3) $-5x^2+4xy+5y^2$

4) $5x^2-28xy+15y^2$

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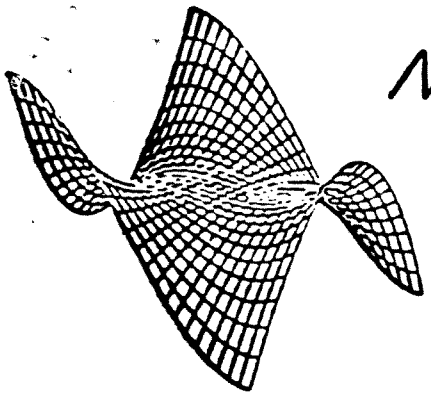


Mathematics Support Capsules

BASIC ALGEBRA

VI. SOLVING EQUATIONS

Principles:

① Addition (& subtraction): if $a = b$, then $a+c = b+c$ for any c.

② Multiplication (& division): if $a=b$, then $ac = bc$, (except division by 0) for any c.

③ Zero product: if $ab = 0$, then $a=0$ or $b=0$.
(Note: the zero on one side is essential; $ab = c \neq 0$ tells you nothing!)

Where do you use these:

→ Many equations can be solved by using ① and ② to isolate x.

(e.g. $3x-5 = 0 \overset{\text{adding 5 to both sides}}{\Longrightarrow} 3x = 5 \overset{\text{dividing both sides by 3}}{\Longrightarrow} x = {}^{5}/_{3}$)

Principles ① and ② tell you that so long as you do something to both sides of an equation, you still have an equation.

→ Many equations in which one cannot isolate x can be solved by ③

(e.g. $x^2 = x+6 \overset{\text{set=0}}{\Longrightarrow} x^2-x-6 = 0 \overset{\text{factor}}{\Longrightarrow} (x-3)(x+2) = 0$

so either factor = 0 : $x-3 = 0$ or $x=3$; $x+2 = 0$ or $x = -2$;

two answers to equation)

→ You get hung up on using ③ if you cannot factor your expression which is =0. However, there is a way out if that expression is quadratic:

④ If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

which is the famous "quadratic formula".

If you have a polynomial of degree higher than quadratic, guess solutions or try to factor, remembering that

⑤ If $x=a$ is a solution, then $x-a$ is a factor.

If none of the above work for your equation,

⑥ Set $f(x) = 0$ and then graph $y = f(x)$.
the solutions to $f(x) = 0$ are where the graph crosses the x-axis.

Exercises

Solve for x:

1) $2x+3 = 0$

2) $(x+7)^2 = x^2-x$

3) $\frac{2}{x} = \frac{x}{8}$

4) $x^2-5x+6 = 0$

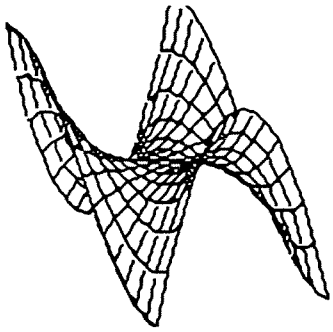
5) $x^2-5x-6 = 0$

6) $x^2+x+6 = 0$

7) $x^3-2x^2+x = 0$

8) solve for a: $ab+c = 3b^2$

9) solve for v_1: $s = \frac{H}{m(v_1-v_2)}$



Mathematics Support Capsules

BASIC ALGEBRA
VII. POST-TEST

Combine and simplify as much as possible the following expressions:

Questions *Answers*

1) $\dfrac{3x}{2x+2y} - \dfrac{x^2}{x+y}$ 1)_____

2) $\dfrac{3x^2}{x+7} \div \dfrac{7x}{x^2+8x+7}$ 2)_____

3) $\dfrac{abc}{ab+cd}$ 3)_____

4) $\dfrac{2a^3}{a^5}$ 4)_____

5) $(-3x^2y)^4$ 5)_____

6) $\dfrac{0.1c^8}{0.5c^6}$ 6)_____

7) $\sqrt{0.04c^6}$ 7)_____

8) $\sqrt[3]{27+x^3}$ 8)_____

9) $(2a+1/a)^3$ 9)_____

10) $(2x+\sqrt{2})(3y-\sqrt{2})$ 10)_____

Solve the following equations for x:

11) $x^3 = 9x^2 - 20x$ 11)_____

12) $2x^2 = 3x - 7$ 12)_____

Check your answers on the next page!

Algebra Post-Test Answers			
Answers to algebra Post-Test	If you need more help: Basic *Algebra* Capsule Sections	Hughes-Hallett, *Algebra*	Asimov, *Realm of Algebra*
1) $\frac{3x-2x^2}{2(x+y)}$ 2) $\frac{3x^2+3x}{7}$ 3) cannot be simplified	II. Fractions	Chapters 4, 10	Chapters 3, 6
4) $\frac{2}{a^2}$ 5) $81x^8y^4$ 6) $0.2c^2$	III. Exponents	Chapter 7	Chapter 8
7) $0.2c^3$ 8) cannot be simplified	IV. Radicals	Chapter 6	Chapter 8
9) $8a^3+12a+6/a+1/a^3$ 10) $6xy-2\sqrt{2}x+3\sqrt{2}y-2$	V. Simplifying Algebraic Expressions	Chapters 3, 8, 9, 11	Chapters 4, 7
11) $x=0,4,5$ 12) $x=\frac{3\pm\sqrt{-47}}{4}$	VI. Solving Equations	Chapters 12, 14, 15	Chapters 5, 8, 9, 10, 11

References above are to:

Isaac Asimov, *Realm of Algebra* (Fawcett Publications, 1961), and

Deborah Hughes-Hallett, *The Math Workshop: Algebra* (W. W. Norton, 1980).

These books (and others) are available for examination or browsing in the Mathematics Support Center and for sale at the Campus Bookstore and other bookstores in Collegetown.